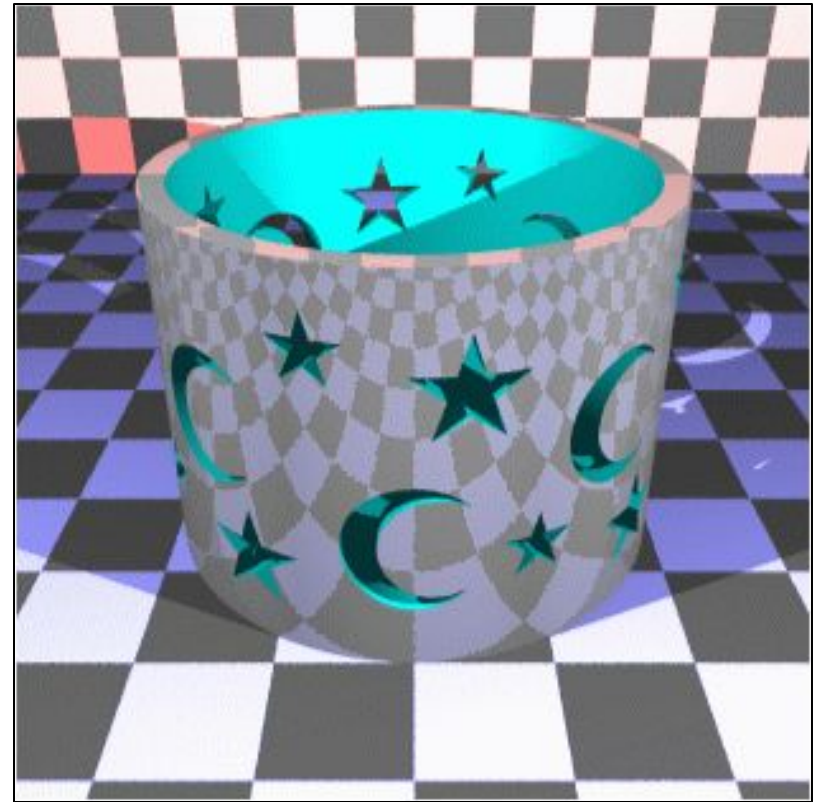


Appendix A:

Constructive Solid Geometry

Constructive Solid Geometry (CSG) is a ray-tracing technique which builds complicated forms out of simple primitives, comparable to (and more complicated than, but also more precise than) Signed Distance Fields.

These primitives are combined with the standard boolean operations: *union*, *intersection*, *difference*.

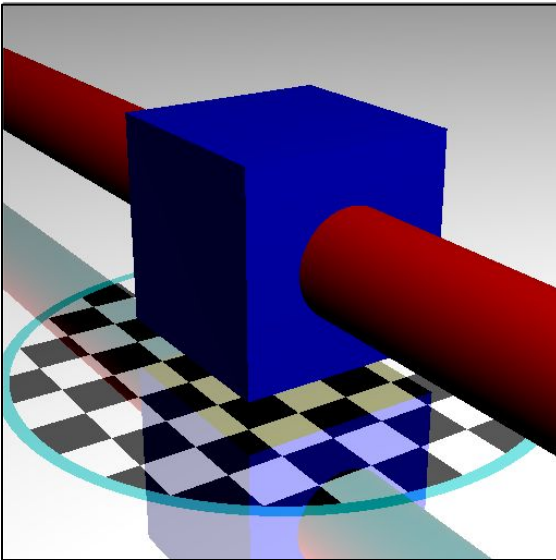


CSG figure by Neil Dodgson

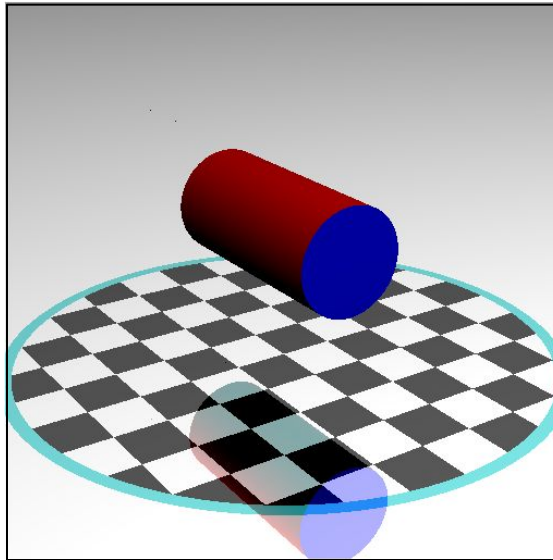
Constructive Solid Geometry

Three operations:

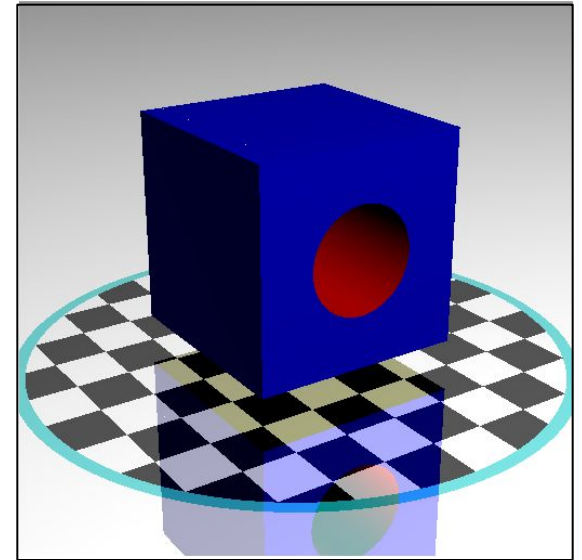
1. *Union*



2. *Intersection*



3. *Difference*



Constructive Solid Geometry

CSG surfaces are described by a binary tree, where each leaf node is a primitive and each non-leaf node is a boolean operation.

(What would the *not* of a surface look like?)

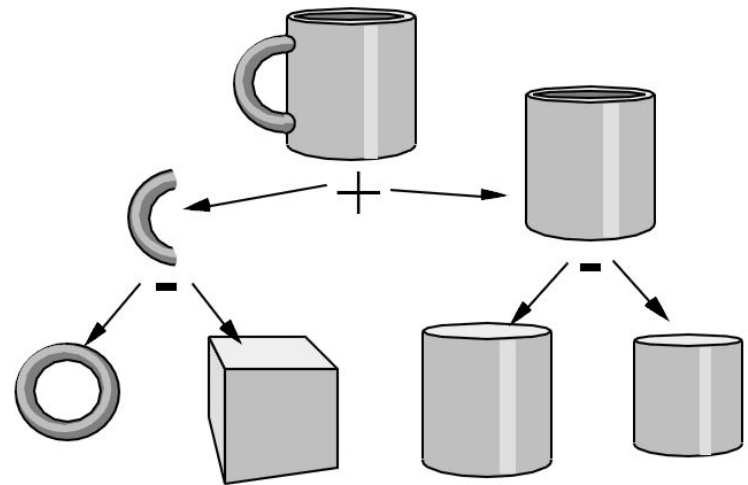
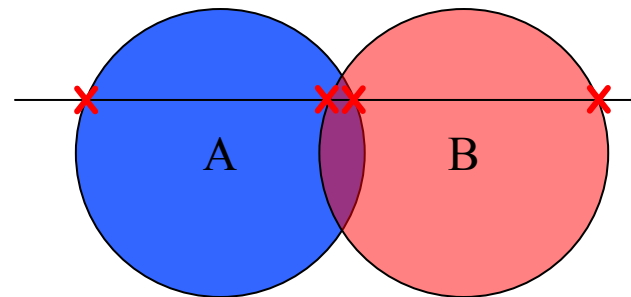


Figure from Wyvill (1995) part two, p. 4

Ray-tracing CSG models

For each node of the binary tree:

- Fire ray r at A and B .
- List in t -order all points where r enters of leaves A or B .
 - You can think of each intersection as a quad of booleans--
($wasInA$, $isInA$, $wasInB$, $isInB$)
- Discard from the list all intersections which don't matter to the current boolean operation.
- Pass the list up to the parent node and recurse.

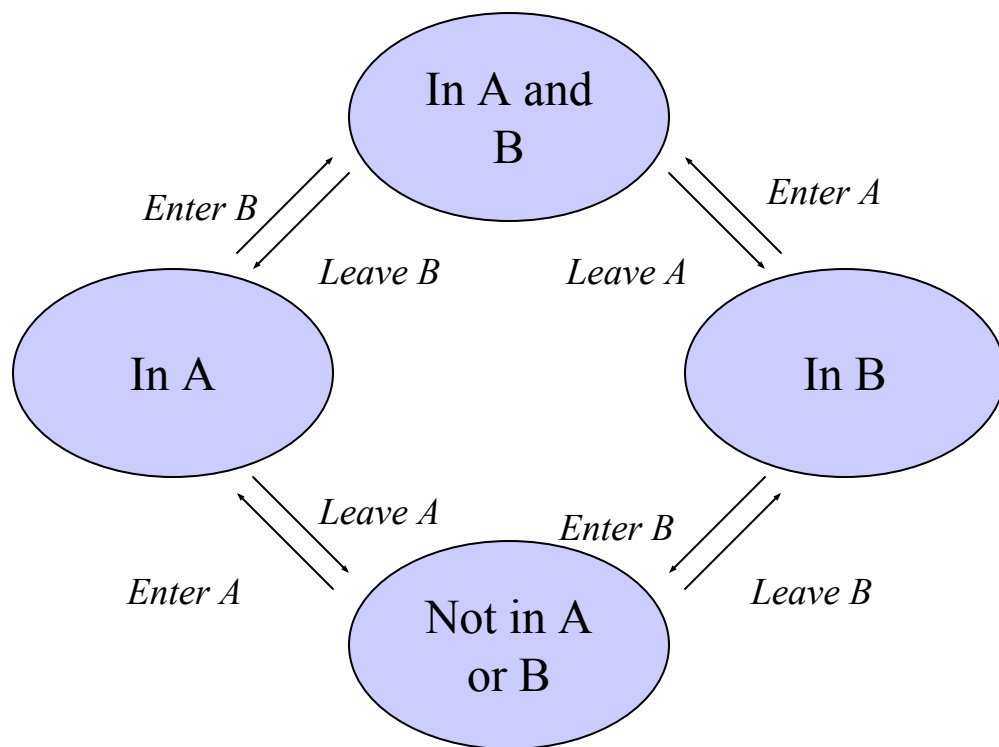


Ray-tracing CSG models

Each boolean operation can be modeled as a state machine.

For each operation, retain those intersections that transition into or out of the critical state(s).

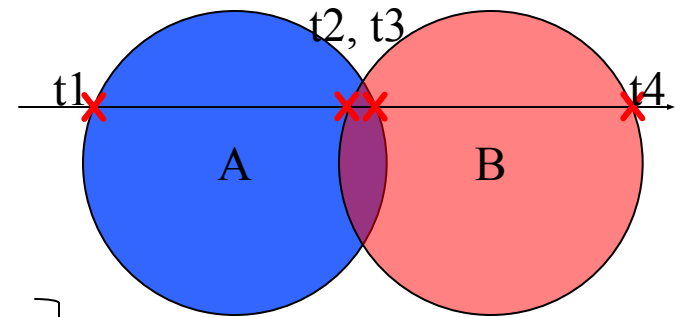
- Union: {In A | In B | In A and B}
- Intersection: {In A and B}
- Difference: {In A}



Ray-tracing CSG models

Example: Difference (A-B)

A-B	Was In A	Is In A	Was In B	Is In B
t1	No	Yes	No	No
t2	Yes	Yes	No	Yes
t3	Yes	No	Yes	Yes
t4	No	No	Yes	No



```
difference =  
( (wasInA != isInA) &&  
  (!isInB) && (!wasInB) )  
||  
( (wasInB != isInB) &&  
  (wasInA || isInA) )
```

Constructive Solid Geometry - References

- Jules Bloomenthal, *Introduction to Implicit Surfaces* (1997)
- Alan Watt, *3D Computer Graphics*, Addison Wesley (2000)
- MIT lecture notes:
<http://groups.csail.mit.edu/graphics/classes/6.837/F98/talecture/>